

Exemple : valeurs et espaces propres p. 137

$$A = \begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_A(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} 2-\lambda & 0 & -3 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(1-\lambda)^2$$

$$E_1 = \ker(A - I_3) = \ker \begin{pmatrix} 1 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 - 3x_3 = 0 \\ x_2 \text{ libre} \\ x_3 \text{ libre} \end{cases}$$

$$\vec{s} = k \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + l \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow E_1 = \text{Span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$E_2 = \ker(A - 2I_3) = \ker \begin{pmatrix} 0 & 0 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_2 = 0 \\ x_3 = 0 \\ x_1 \text{ libre} \end{cases}$$

$$\Rightarrow E_2 = \text{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$